Centre for Process Systems Engineering

Imperial College London

Advances in multi-parametric programming & explicit Model Predictive Control

Stratos Pistikopoulos

Session in honor of Professor Ignacio E. Grossmann

Outline

- Multi-parametric programming & control – *how it started [with Ignacio – of course!]*
- Multi-parametric programming & control – *brief status report*
- Focus on
  - mp-MIQP [*the exact solution*]
  - The PAROC framework & software

Professor Ignacio E. Grossmann

Or

Wise Men Sense Things about to Happen

For the gods know the future, men the present, but wise men that which is approaching

(Philostratos, On Apollonios of Tyana, viii, 7)

How it started

- Work on flexibility w/Ignacio (1984-1988)
  - active sets
  - explicit expressions fro feasibility/flexibility

How it started

- Work on flexibility w/Ignacio - for example, Pistikopoulos & Grossmann – appendix c (1988)

OPTIMAL RETROFIT DESIGN FOR IMPROVING PROCESS FLEXIBILITY IN LINEAR SYSTEMS

E. N. PISTIKOPOULOS and I. E. GROSSMANN

Computers & Chemical Engineering, *Volume 12, Issue 7*, *July 1988*, *Pages 719-731*

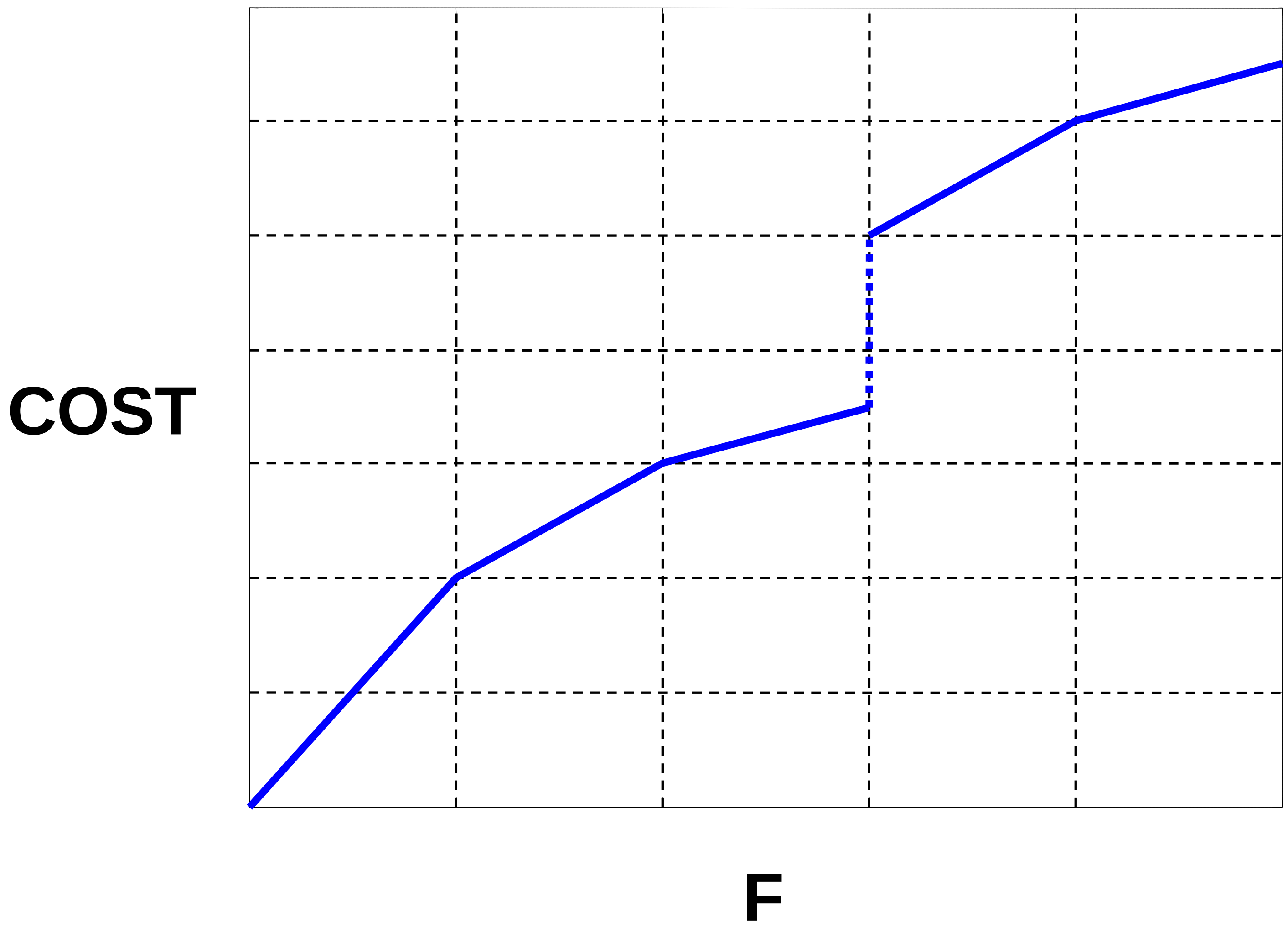
Abstract--In this paper the problem of optimally redesigning an existing process to increase its flexibility is addressed. Assuming a linear model for the process, a general strategy is proposed which determines first the optimal parametric changes and then identifies the optimal structural modifications. Basic analytical properties of flexibility are presented with which very efficient reduced LP and MILP formulations can be developed that include explicit constraints for flexibility. Also, trade-off curves relating flexibility to retrofit cost can easily be generated to provide information on the cost of flexibility. Examples are presented to illustrate the proposed procedures.

Appendix c - Trade-off Curve for Cost vs Flexibility--MILP Case

Appendix c - Trade-off Curve for Cost vs Flexibility--MILP Case

In this appendix, it will be shown how the trade-off curve of cost vs flexibility can be obtained when fixed cost charges are included. A typical trade-off curve for this MILP problem is shown in Figure C1. It exhibits two important features: (a) it is discontinuous at the break points defined by the change of the limiting active sets; (b) it might be piecewise continuous within the region characterized by the same limiting active sets due to a change of the design variables to be modified.

The above features suggest that for generating the MILP trade-off curve it suffices to identify the points of discontinuity (i.e. change of the limiting active sets), as well as the possible break points within the region associated to a given active set.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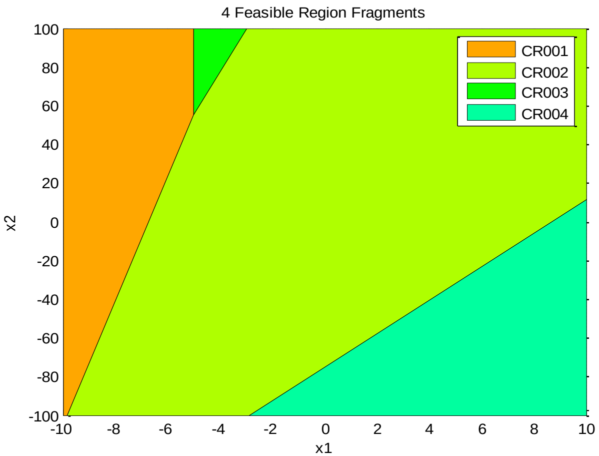
Multi-parametric programming

$$\min_u \left(-3u_1 - 8u_2\right)$$

st.

$$\begin{bmatrix} 1 & 1 \\ 5 & -4 \\ -8 & 22 \\ -4 & -1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -13 \\ -20 \\ -121 \\ 8 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

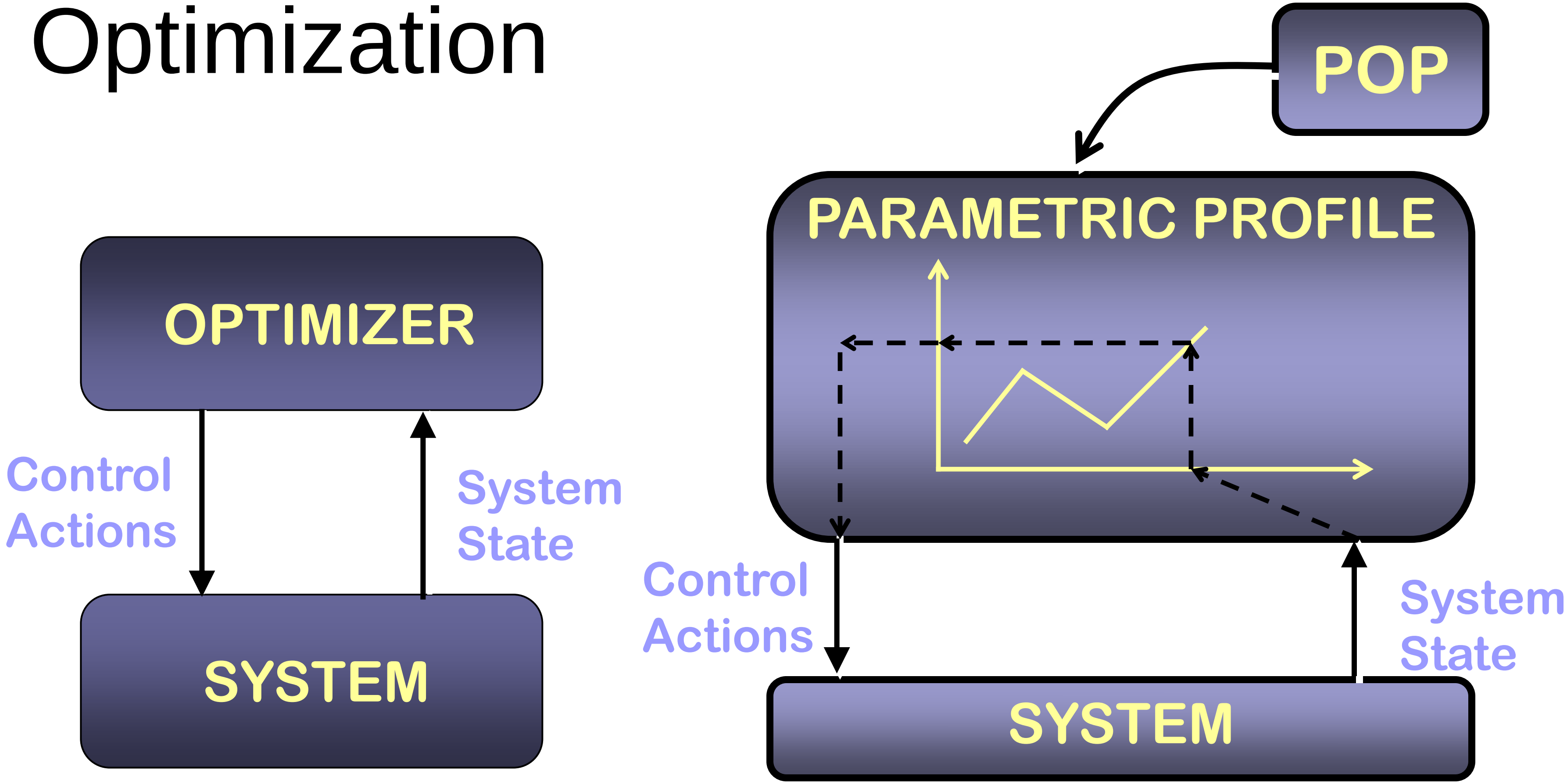
$$-10 \leq x_1 \leq 10, -100 \leq x_2 \leq 100$$



$$U = \begin{cases} \begin{bmatrix} -0.333 & 0 \\ 1.333 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1.6667 \\ 14.6667 \end{bmatrix} & \textbf{if} \quad \begin{bmatrix} 1 & -0.03125 \\ 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} -6.71875 \\ -5 \\ 10 \\ 100 \\ 100 \end{bmatrix} \\ \begin{bmatrix} 0.7333 & -0.0333 \\ 0.26667 & 0.03333 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5.5 \\ 7.5 \end{bmatrix} & \textbf{if} \quad \begin{bmatrix} 1 & -0.115385 \\ -1 & 0.03125 \\ -1 & 0.0454545 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 8.65385 \\ 6.71875 \\ 7.5 \\ 10 \\ 100 \\ 100 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 13 \end{bmatrix} & \textbf{if} \quad \begin{bmatrix} 1 & -0.0454545 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} -7.5 \\ 5 \\ 100 \end{bmatrix} \\ \begin{bmatrix} 0 & 0.05128 \\ 0 & 0.0641 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 11.8462 \\ 9.80769 \end{bmatrix} & \textbf{if} \quad \begin{bmatrix} -1 & 0.115385 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} -8.65385 \\ 10 \\ 100 \end{bmatrix} \end{cases}$$

Only 4 optimization problems solved!

On-line Optimization via off-line Optimization



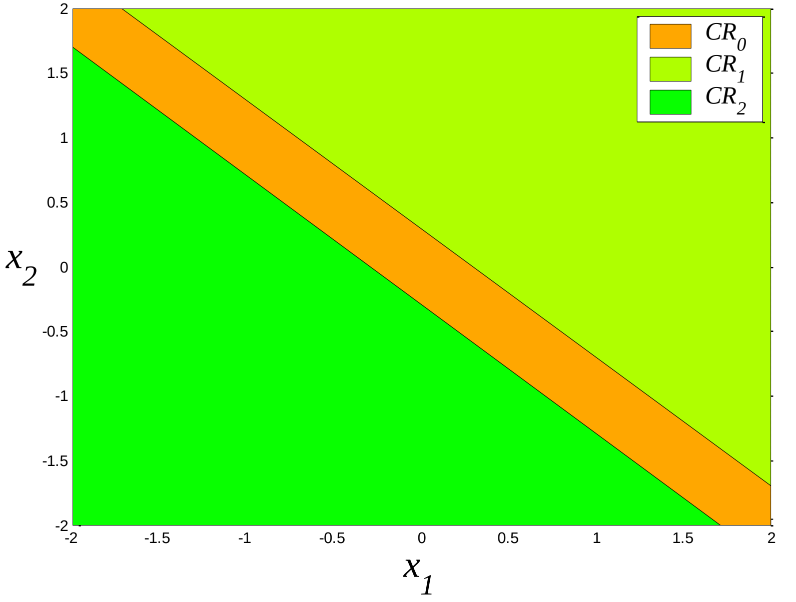
Function Evaluation!

Explicit Control Law

$$J(x(t)) = \min_{u_{t|t},u_{t+1|t}} \sum_{j=0}^{1} \left\{ \mathbf{x}_{t+j|t}^T \mathbf{x}_{t+j|t} + 0.01\,\mathbf{u}_{t+j|t}^2 \right\} + \mathbf{x}_{t+2|t}^T P\,\mathbf{x}_{t+2|t}$$

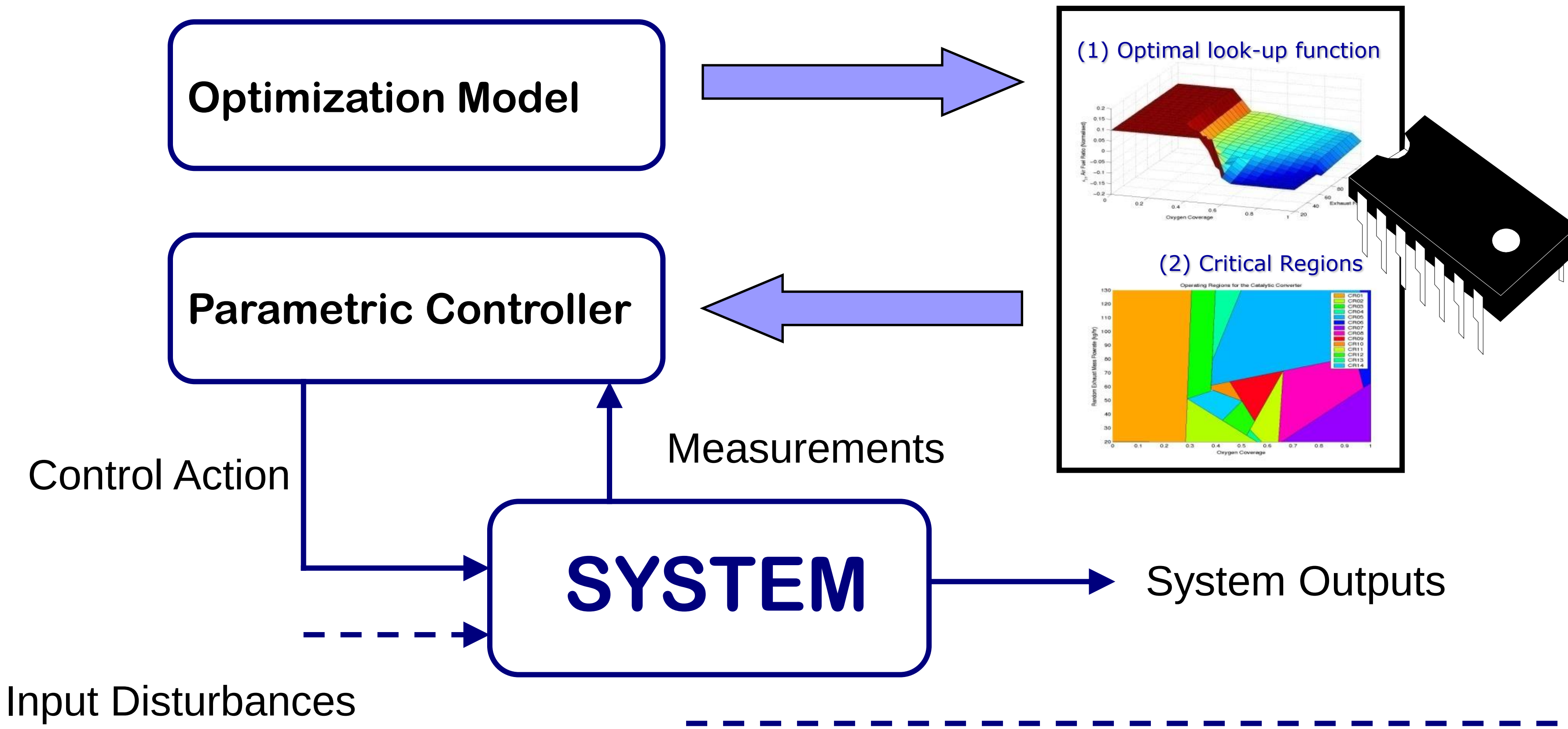
$$\text{s.t} \quad \mathbf{x}_{t+j+1|t} = \begin{bmatrix} 0.7326 & -0.0861 \\ 0.1722 & 0.9909 \end{bmatrix} \mathbf{x}_{t+j|t} + \begin{bmatrix} 0.0609 \\ 0.0064 \end{bmatrix} \mathbf{u}_{t+j|t}$$

$$-2 \le \mathbf{u}_{t+j|t} \le 2 \quad \mathrm{j}=1,2 \quad \mathbf{x}_{t|t} = \mathbf{x}(t)$$



$$\mathbf{u}(t) = \begin{cases} [-6.8355 \;\; -6.8585]\,\mathbf{x}(t) & \textbf{if} \begin{bmatrix} 0.7059 & 0.7083 \\ -0.7059 & -0.7083 \end{bmatrix} \mathbf{x}(t) \le \begin{bmatrix} 0.2065 \\ 0.2065 \end{bmatrix} \\ -2 & \textbf{if} \; [-0.7059 \;\; -0.7083]\,\mathbf{x}(t) \le -0.2065 \\ 2 & \textbf{if} \; [0.7059 \;\; 0.7083]\,\mathbf{x}(t) \le -0.2065 \end{cases}$$

Multi-parametric Controllers



MPC-on-a-chip!

- **Explicit Control Law**
- **Eliminate expensive, on-line computations**
- **Valuable insights !**

Multi-parametric programming & Model Predictive Control [MPC]

- Theory of multi-parametric programming
  - Multi-parametric mixed integer quadratic programming [mp-MIQP]
  - Multi-parametric dynamic optimization [continuous-time, mp-DO]
  - Multi-parametric global optimization
- Theory of multi-parametric/explicit model predictive control [mp-MPC]
  - Explicit robust MPC of hybrid systems
  - Explicit MPC of continuous time-varying [dynamic] systems
  - Explicit MPC of periodic systems
  - Moving Horizon Estimation & mp-MPC

Multi-parametric programming & Model Predictive Control [MPC] – cont'd

- Framework for multi-parametric programming & control
  - Model approximation [from high fidelity models to the design of explicit MPC controllers]
  - Software development, prototype & demonstrations [for teaching & research]
- Application areas
  - Fuel cell energy system – experimental/laboratory
  - Car system control – prototypes/laboratory
  - Energy systems [CHP and micro-CHP]
  - Bio-processing [continuous production & control of monoclonal antibodies]
  - Pressure Swing Absorption [PSA] and hybrid systems
  - Biomedical Systems

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mp-MIQP – Problem Formulation

$$
\begin{aligned}
z(\theta) = \min_{x,y} \quad & (Q\omega + H\theta + c)^T \omega \\
\text{s.t.} \quad & Ax + Ey \le b + F\theta \\
& x \in \mathbb{R}^n, \quad y \in \{0,1\}^p, \quad \omega = \left[x^T \; y^T\right]^T \\
& \theta \in \Theta = \{\theta \in \mathbb{R}^q \,|\, \theta_l^{min} \le \theta_l \le \theta_l^{max}, l = 1, \ldots, q\},
\end{aligned}
$$

Nonconvex critical regions!

For example:

$$
Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; H = \begin{bmatrix} 5 & -2 \\ 0 & -8 \\ -2 & 3 \\ -3 & -1 \end{bmatrix}, \; c = \begin{bmatrix} 0 \\ 0 \\ -5 \\ 0 \end{bmatrix}
$$

$$
A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \; E = \begin{bmatrix} 0 & -1 \\ -1 & 2 \\ 3 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; b = \begin{bmatrix} 1.5 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \; F = \begin{bmatrix} 0 & 2 \\ 2 & 0 \\ 2 & 0.5 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}
$$

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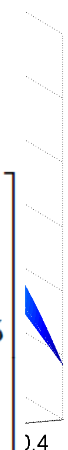


mp-MIQP Problems – Earlier Approaches

Dua et al. (2002):	Decomposition-type approach
	No comparison of objective function
	Enclosure of solutions
Axehill et al. (2014):	Branch-and-bound approach
	Comparison over entire critical region
	Enclosure of solutions
Oberdieck et al. (2014):	Branch-and-bound approach
	Direct comparison using linearization via McCormick relaxations (1976)
	Enclosure of solutions
This approach	Applicable to branch-and-bound and decomposition-type approaches
	Piecewise affine approximation of critical region using McCormick relaxations (1976)
	Exact solution

Applications - hybrid control, reactive/proactive scheduling under uncertainty, etc

mp-MIQP Problems – The exact solution

Comparison of quadratic objective functions → Nonconvex critical regions → Global Optimization

Step 1

Create piecewise affine McCormick relaxations for quadratic boundaries

Step 2

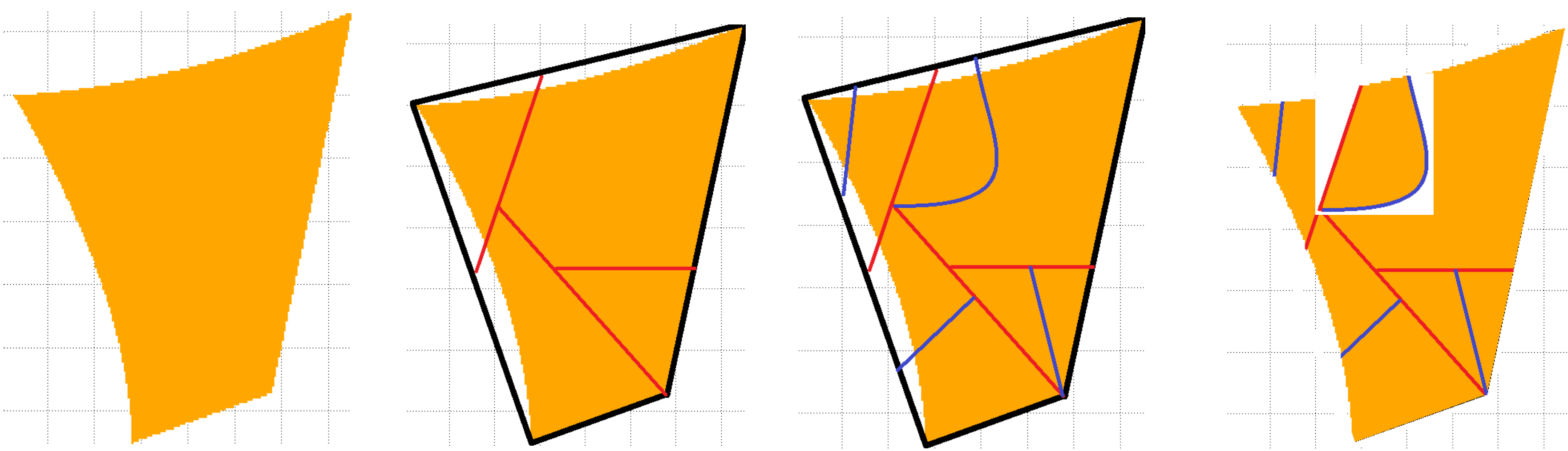
Solve mp-QP in affine space

Step 3

Compare with upper bound

Step 4

Re-introduce original nonaffine constraints and continue with next CR



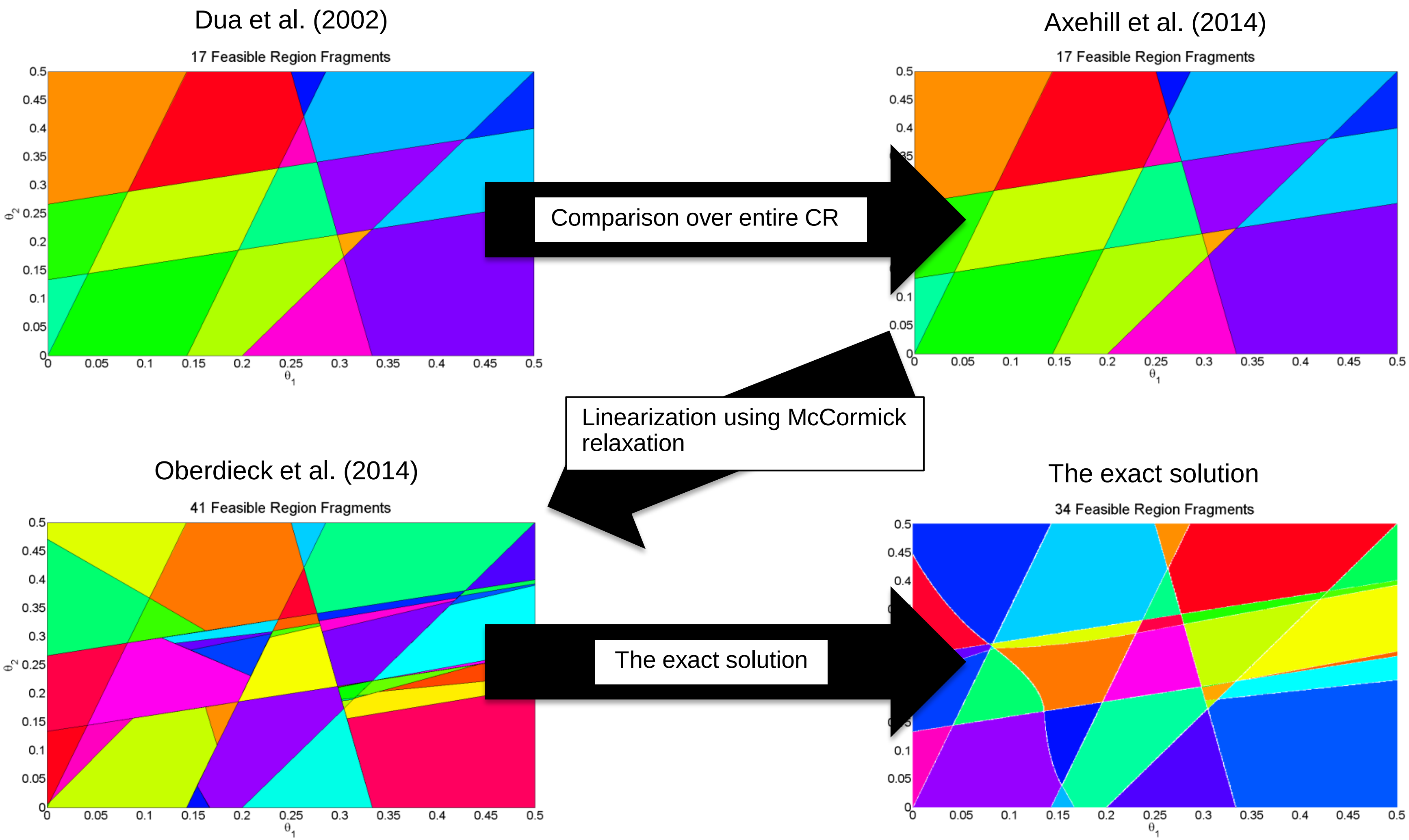
mp-MIQP Problems – Computational Results

Procedure	CPU Time (s)	# CRs	Avg. sol/CR
None (used in Dua et al., 2002)	0.7610	17	3.412
MinMax (Axehill et al., 2014)	1.0126	17	2.059
Affine (Oberdieck et al., 2014)	1.6646	41	1.268
Exact Solution (Oberdieck and Pistikopoulos, 2014)	1.4637	34	1

Similar computational times

Lowest (possible) number of solutions per CR

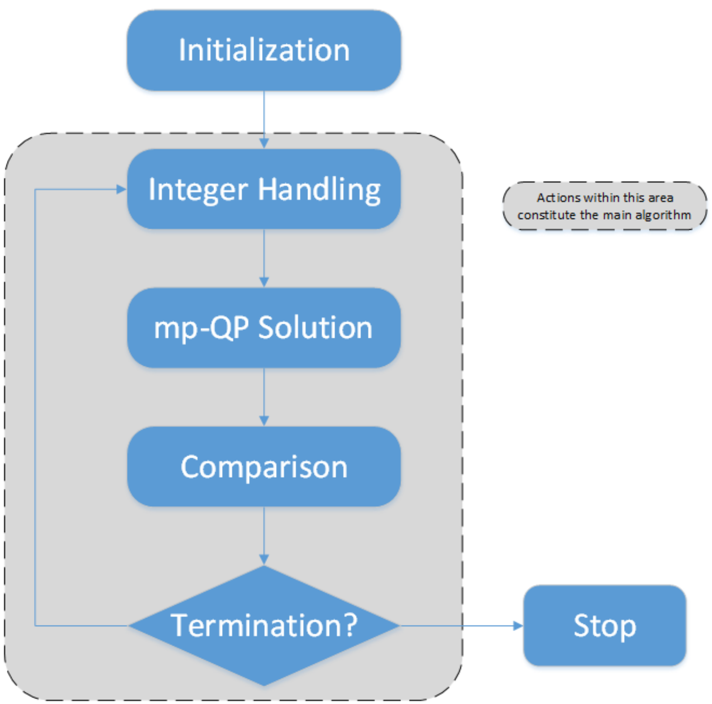
mp-MILP problems – Comparisons

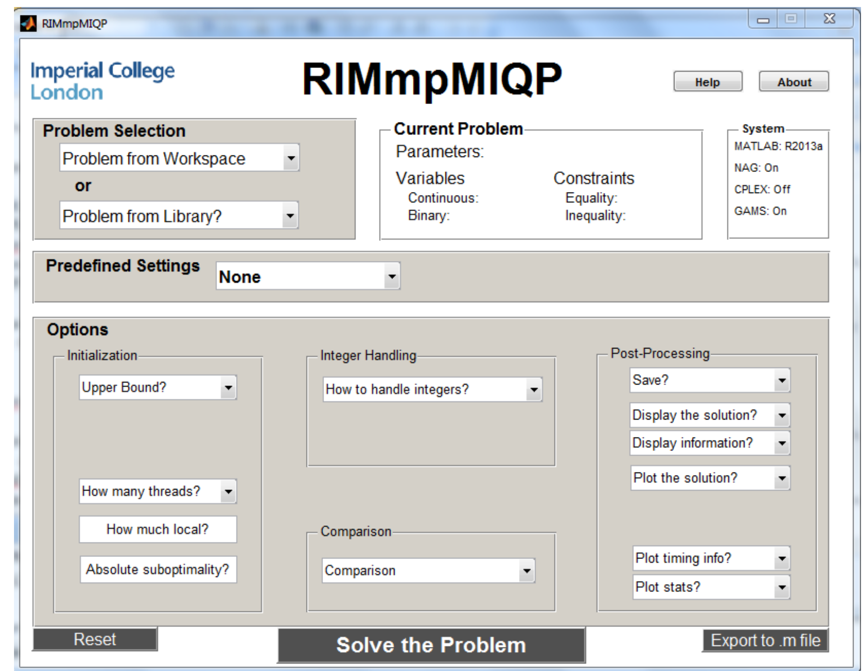


mp-MIQP Problems – Solution platform

- All approaches follow a general framework

This has led to a software solution





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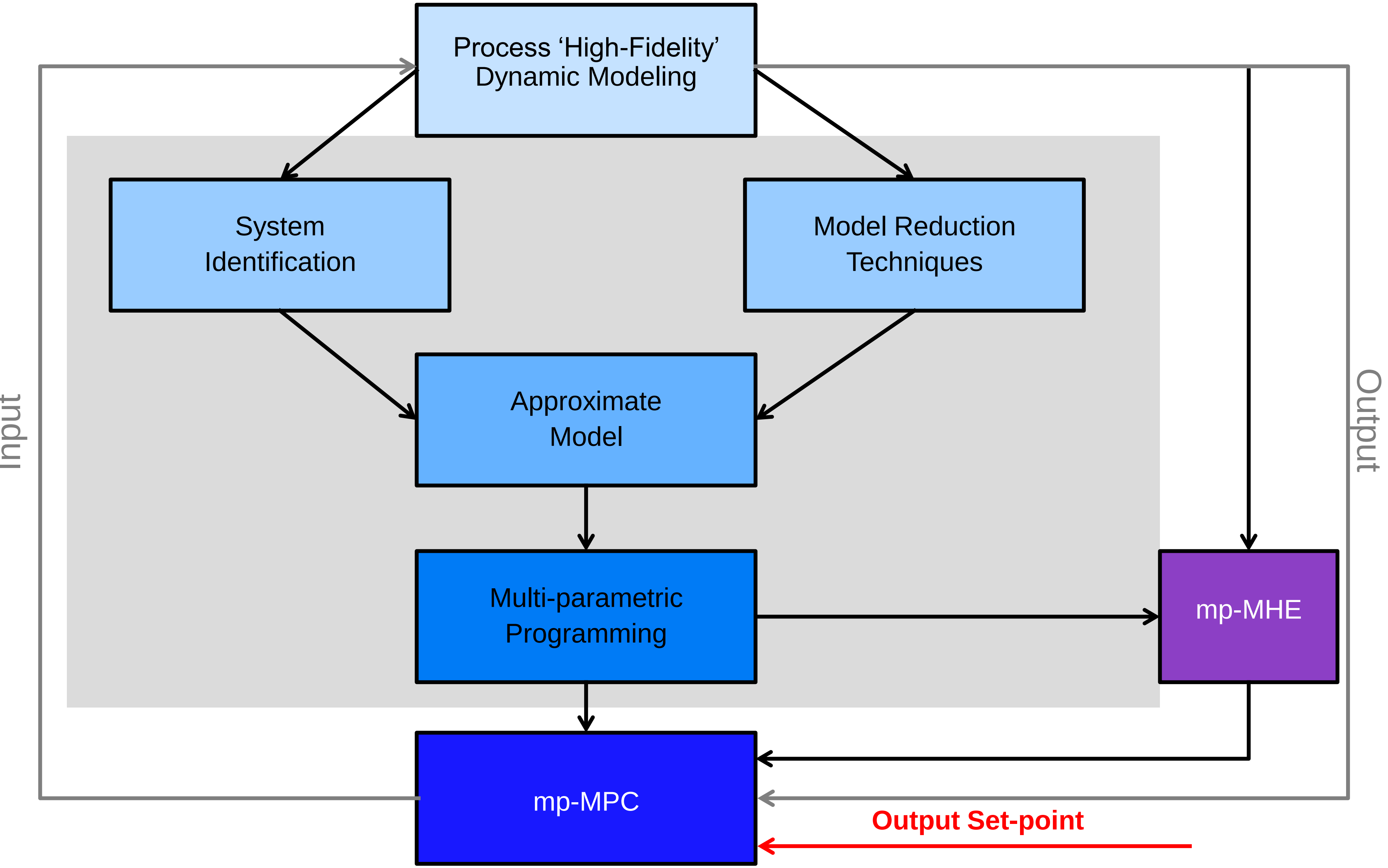
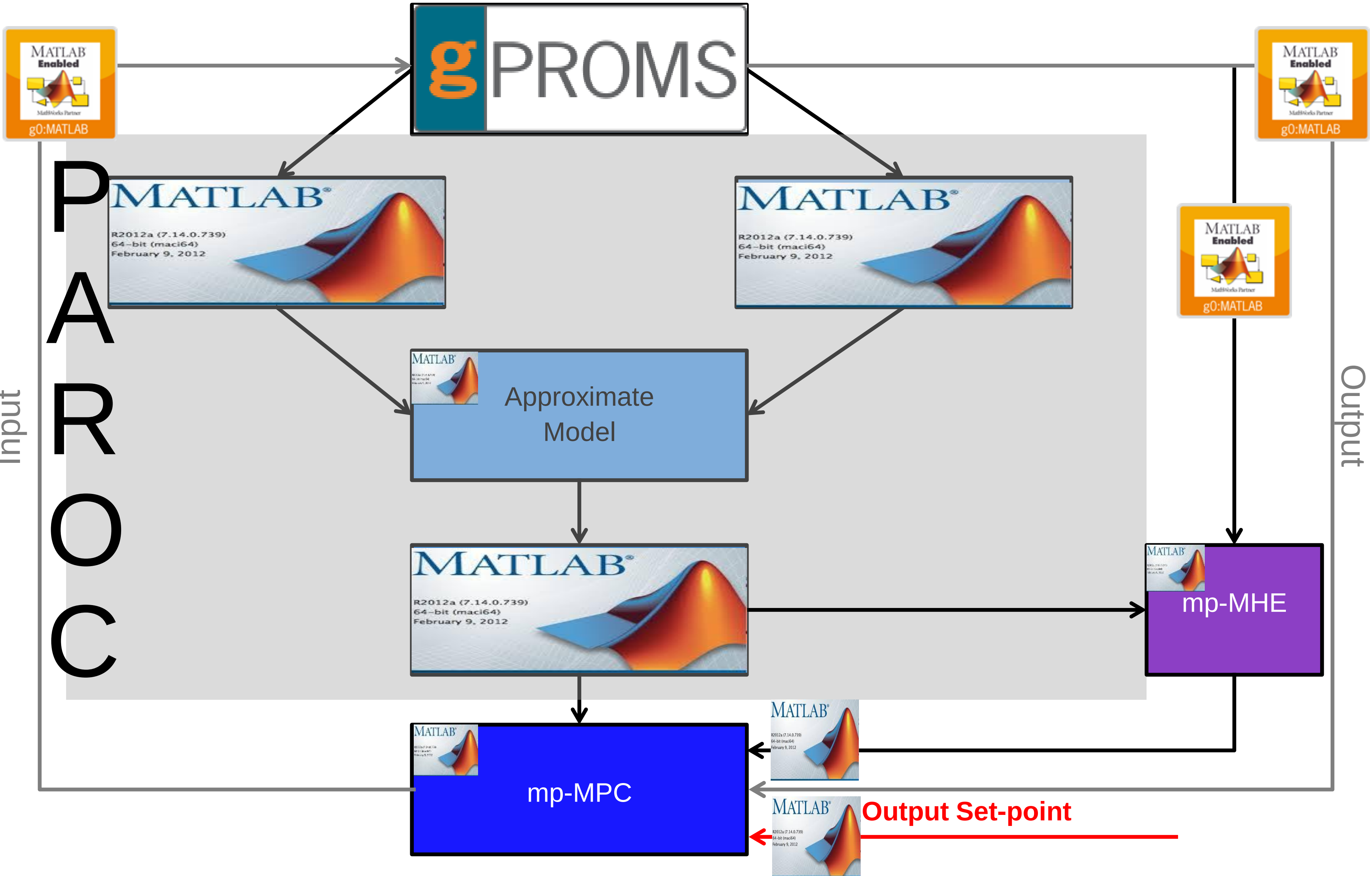
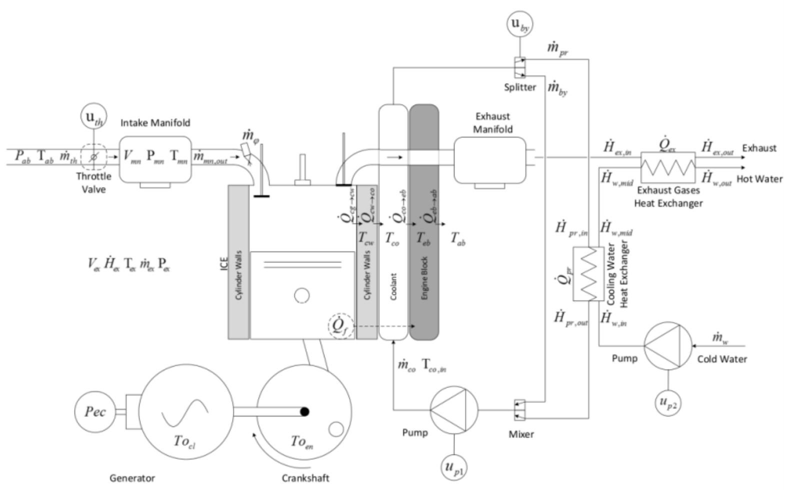
PAROC Framework

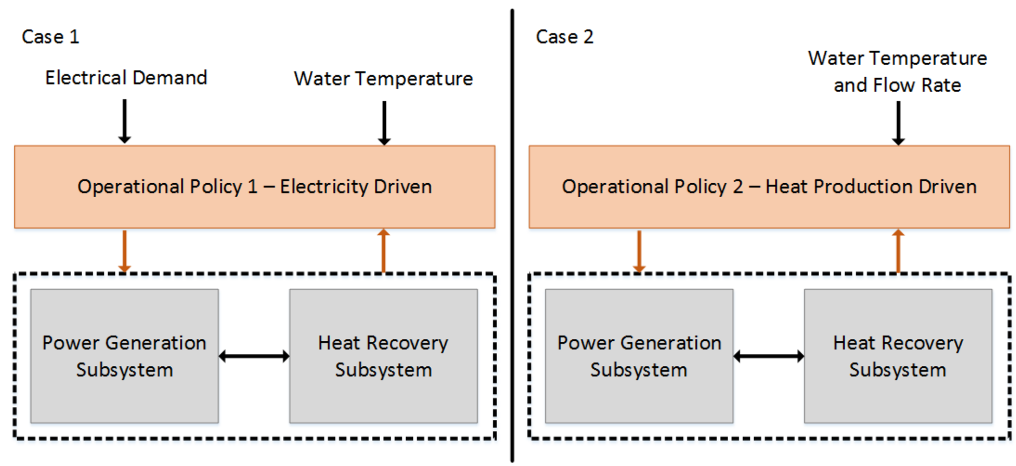


PAROC Framework - *Software*



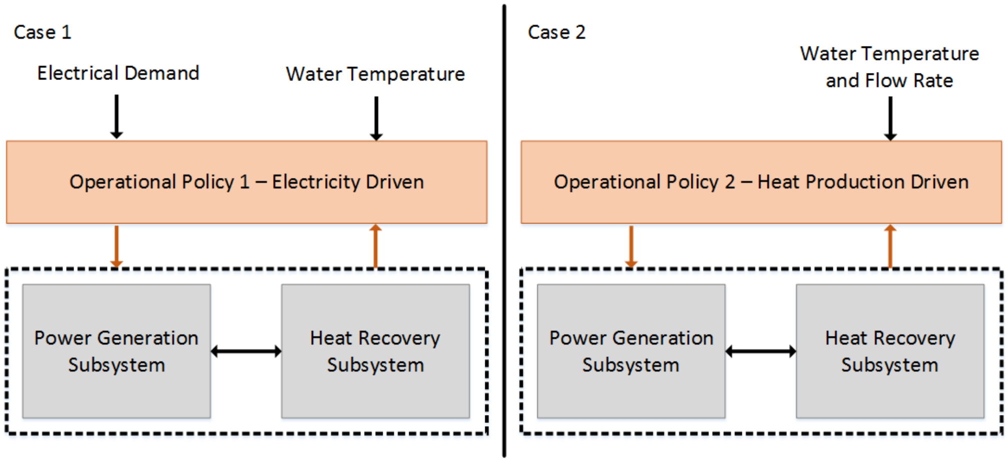
PAROC Framework – *CHP*

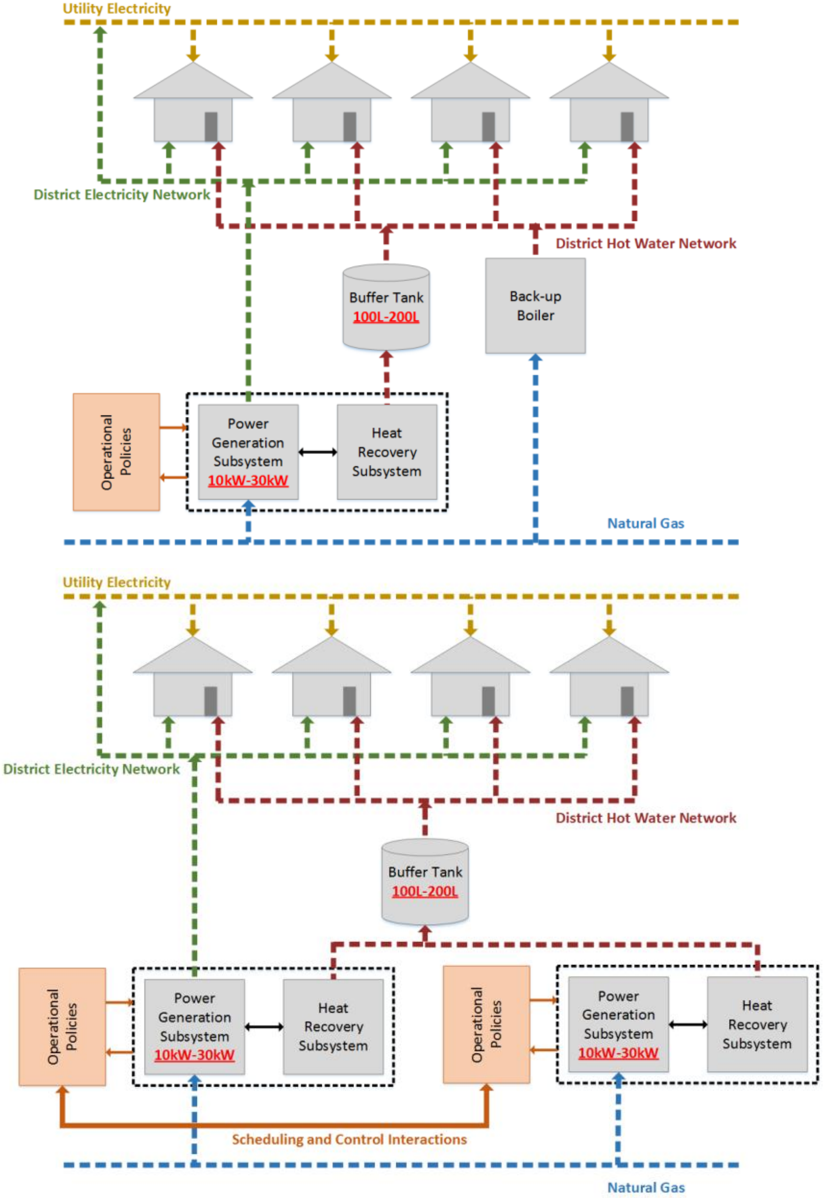




- Cogeneration of heat and power production – trade offs
- Design and control interactions

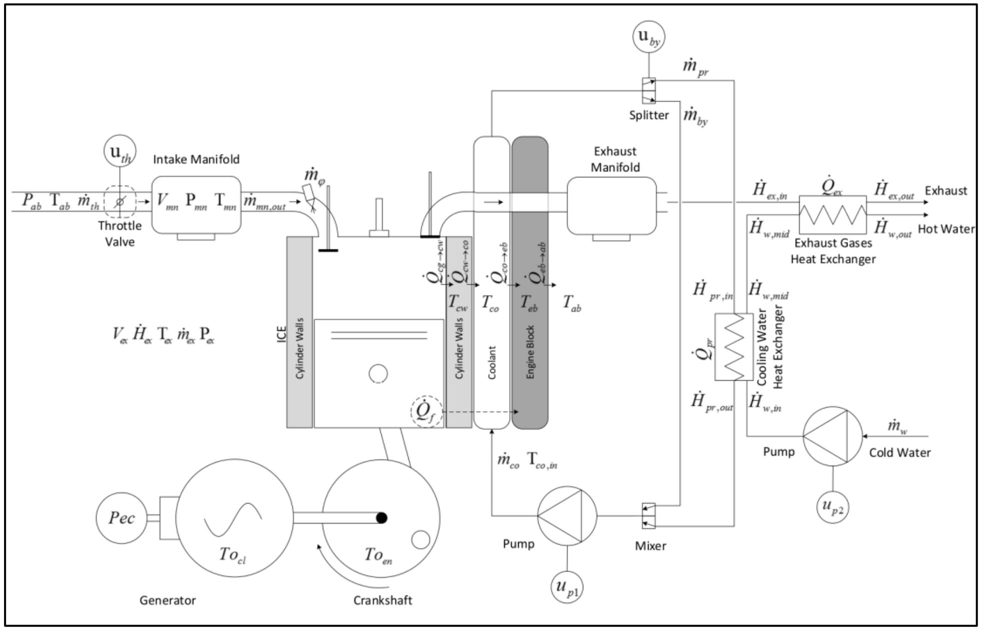
PAROC Framework – *CHP*



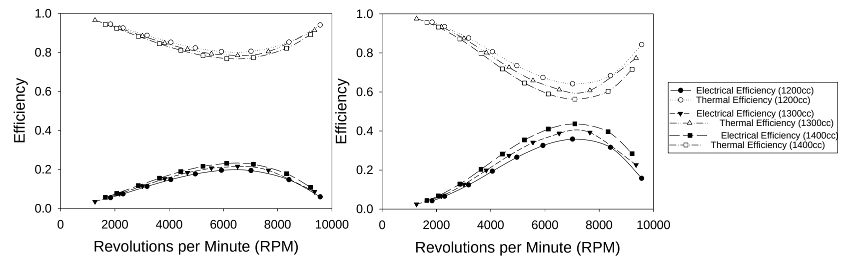


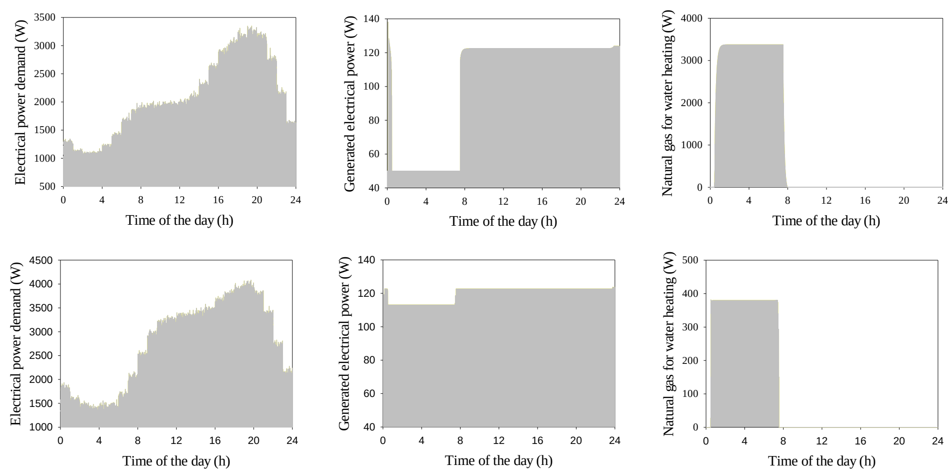
- Cogeneration of heat and power production – trade offs
- Design and control interactions
- Scheduling and control interactions
- Unified modelling representation
- Grand unification of design scheduling and control

CHP - Dynamic modeling and model validation



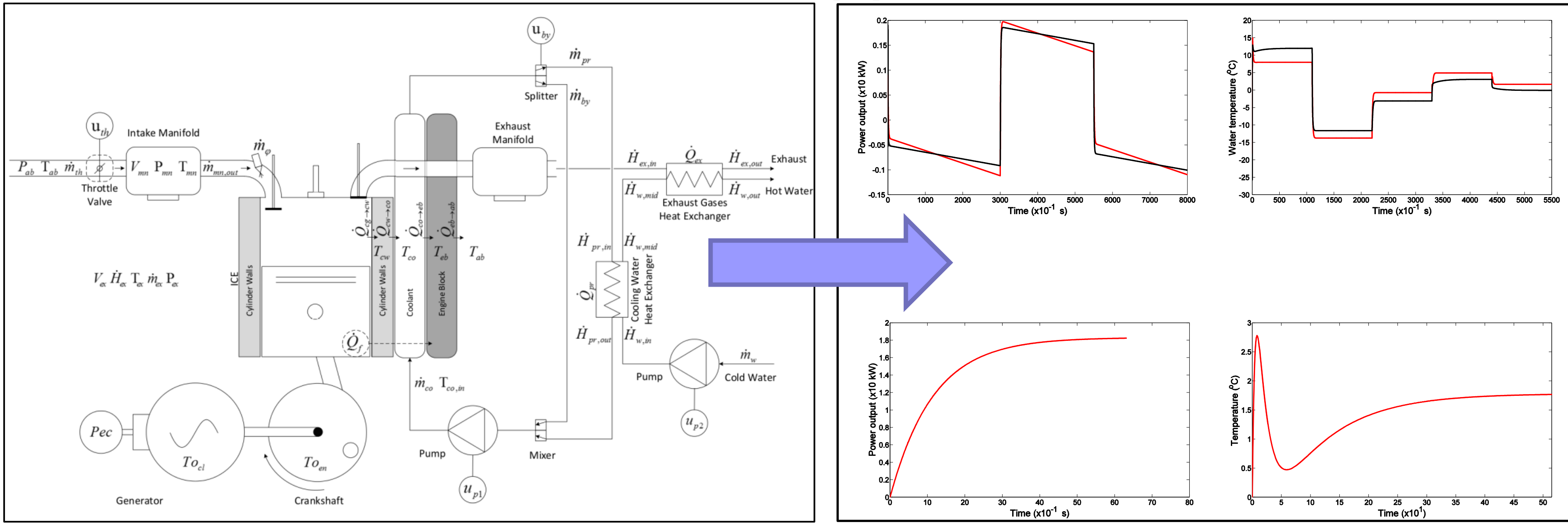
Simulation, validation and dynamic optimisation results





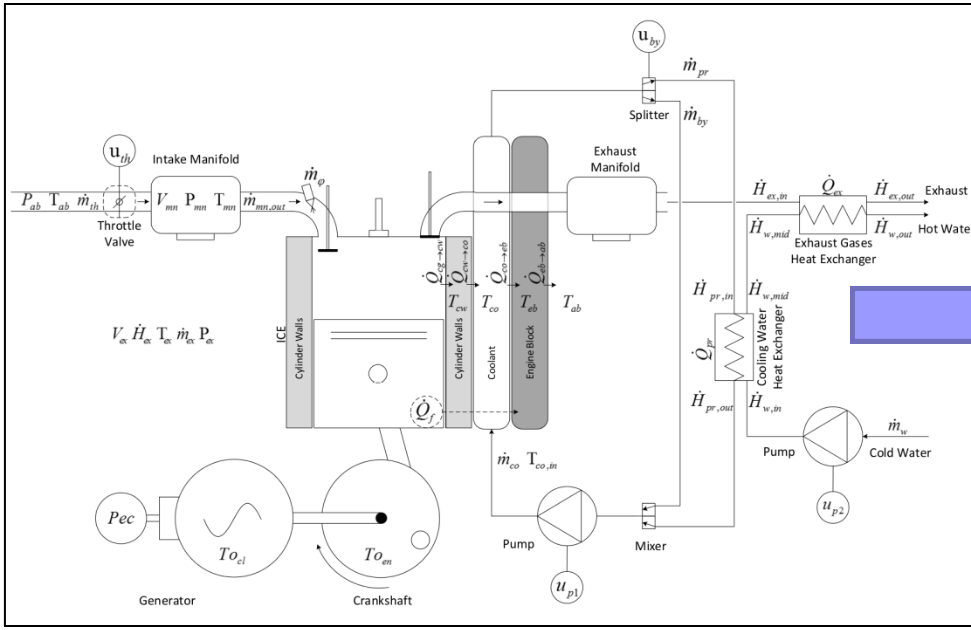
- Complex DAE System
  - 379 Model Equations
  - 15 Differential Equations
  - 4 Degrees of Freedom
- 2 Design variables
- 2 Operational set-points

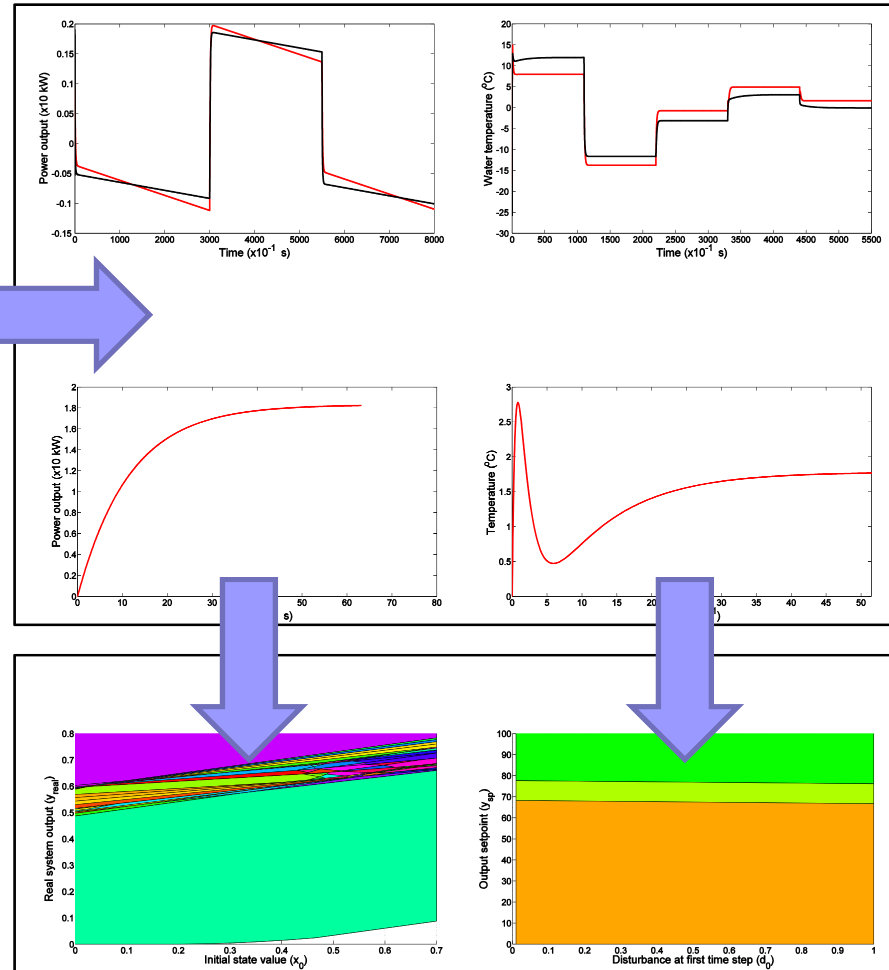
CHP - Model Approximation



- Model Approximation via MATLAB® System Identification Toolbox
  - Identification of two state-space models
  - SISO Power generation subsystem with design variable dependency treated as uncertainty
  - SISO Heat Recovery subsystem with measured disturbance and design variable dependency treated as uncertainty
- Design optimization dependent state-space models
- Scheduling optimisation dependent production set-points

CHP - Receding horizon policies

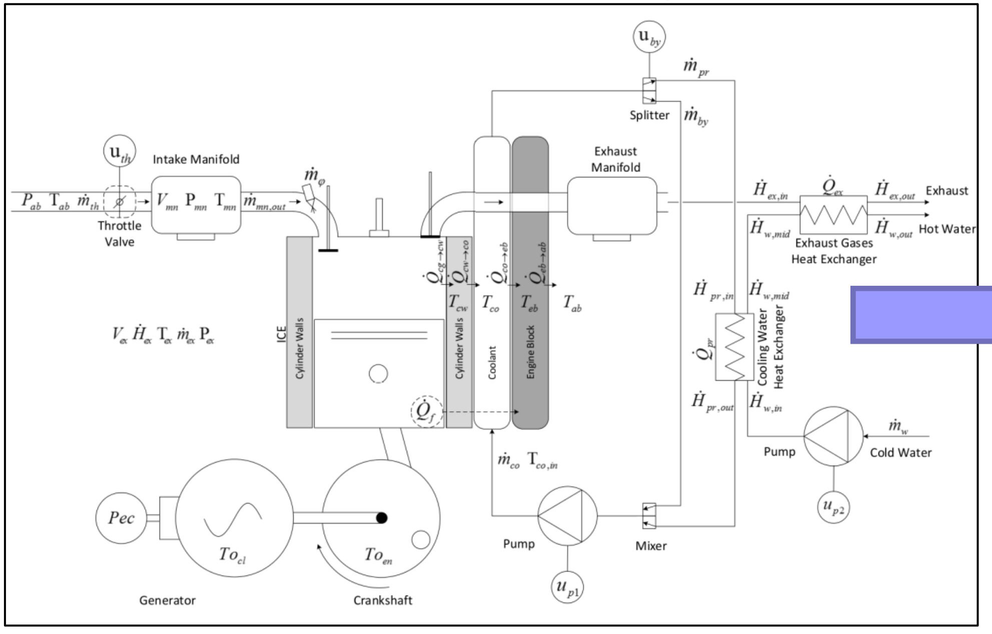


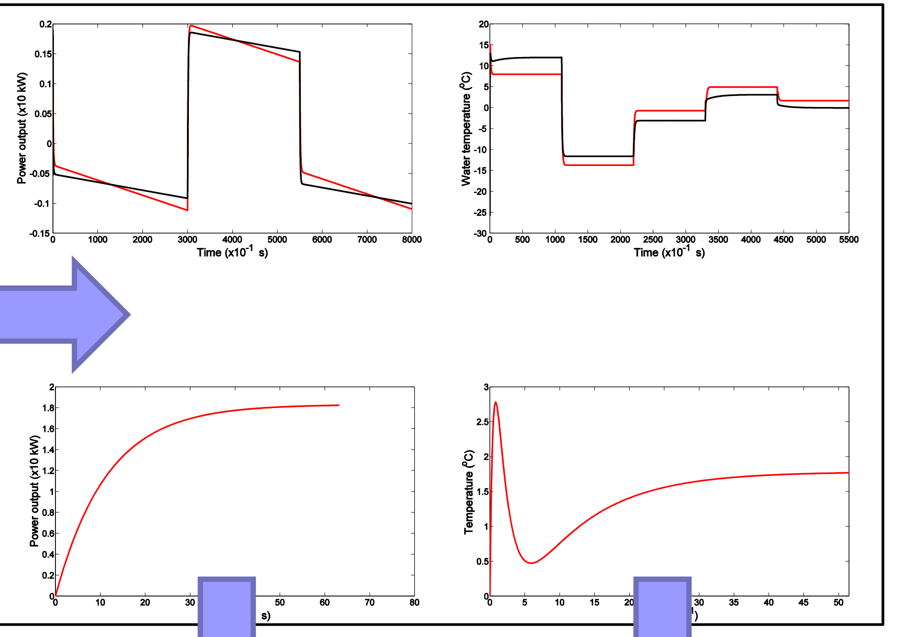


- Decentralised mp-MPC to tackle contradictory objectives
- Realisation of the inherent design uncertainty via dynamic optimization
- Optimal scheduling-dependent output set-points

mp-P receding horizon policies with optimal design and scheduling dependence

CHP - Receding horizon policy implementation





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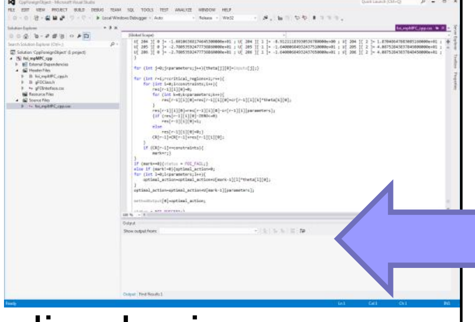
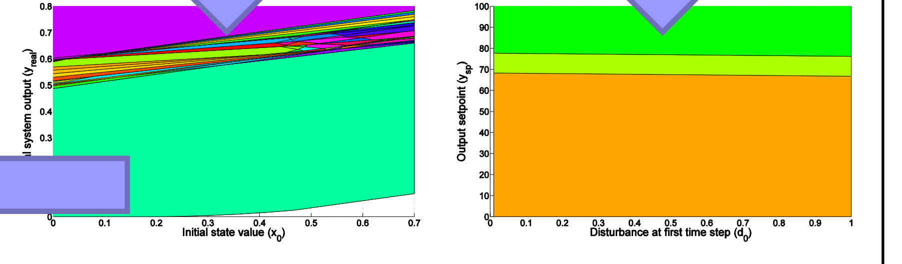
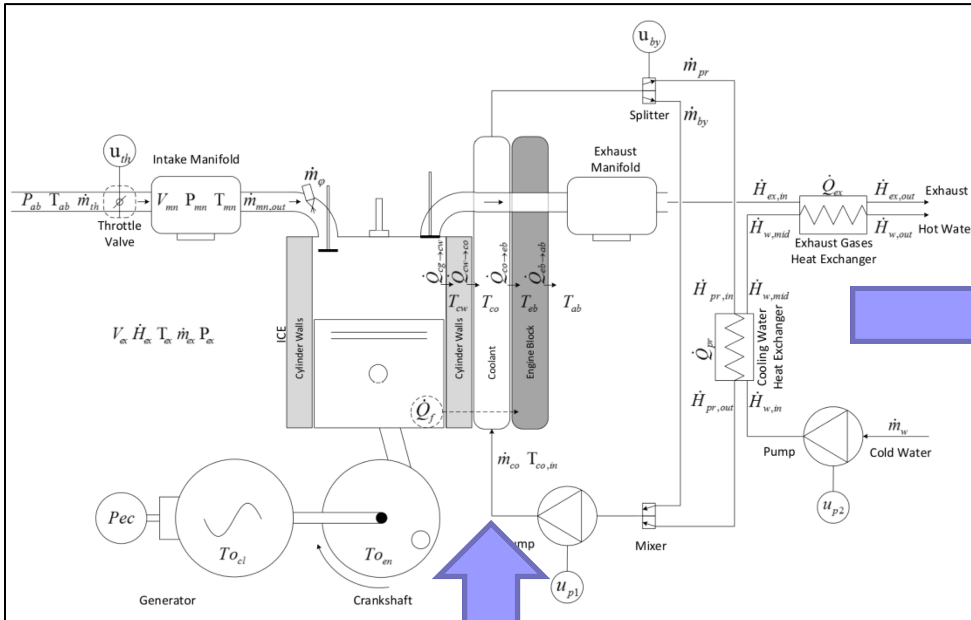


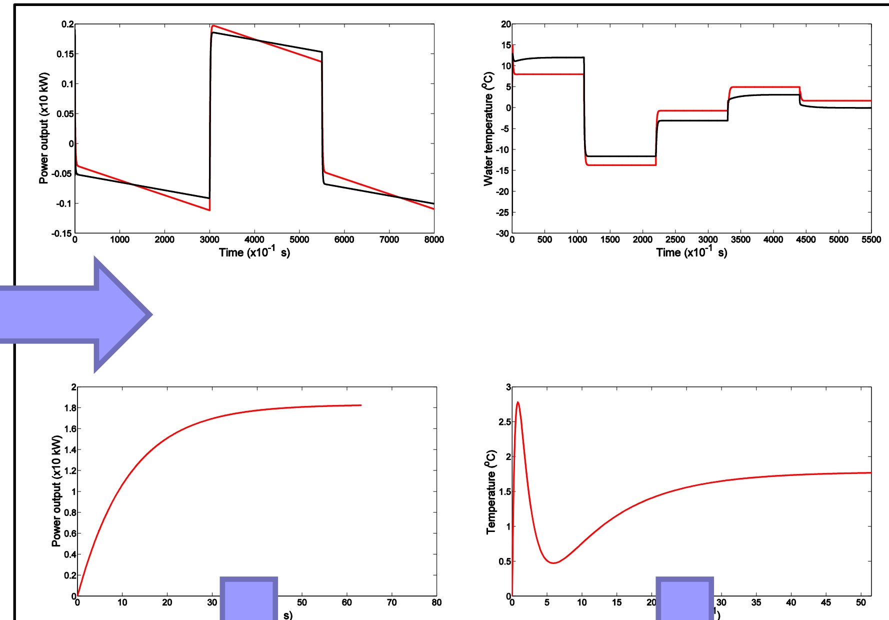
Integration of the receding horizon policies in the model via dynamic link library development or gO:MATLAB®

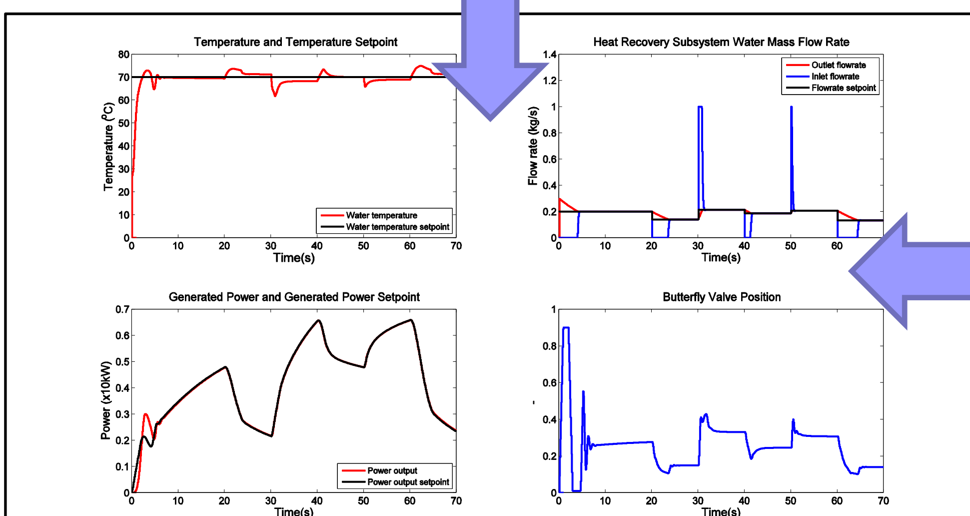


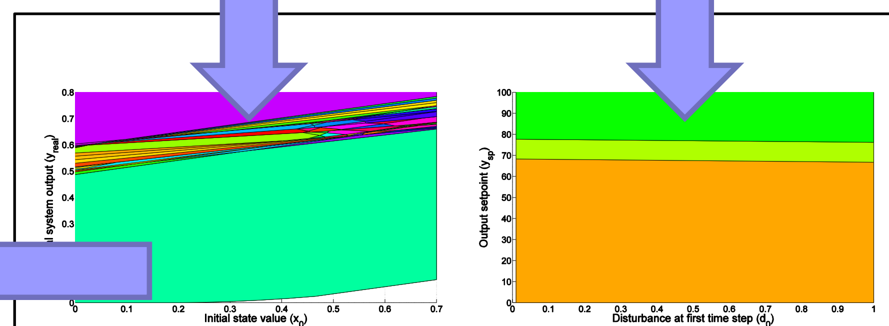
mp-P receding horizon policies with optimal design and scheduling dependence

CHP - Closed-loop validation



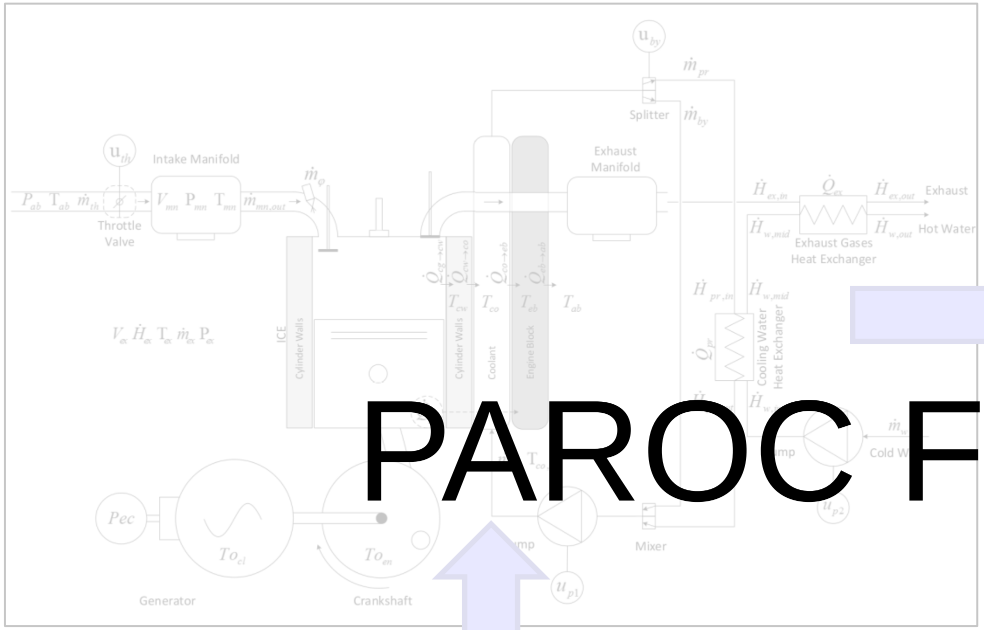


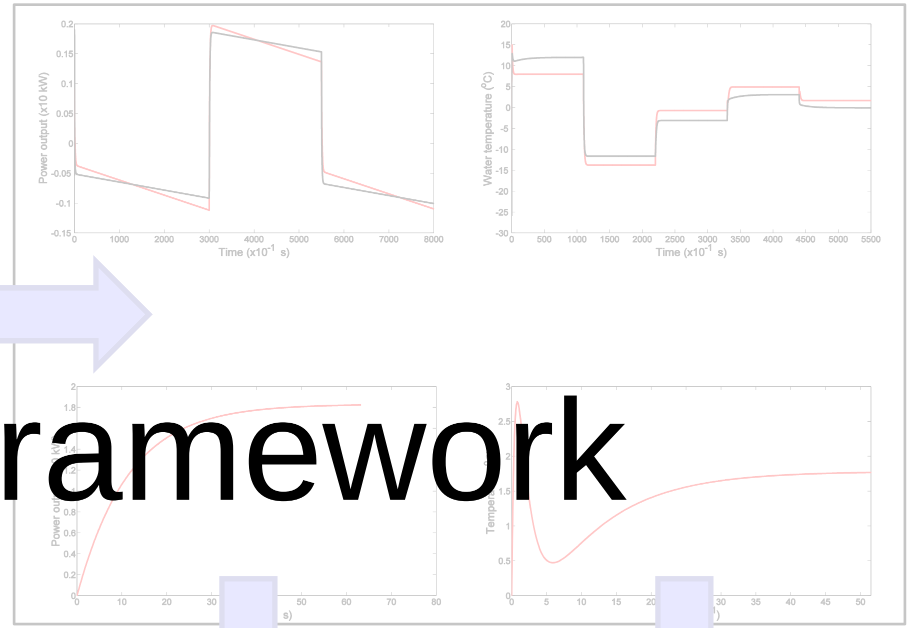


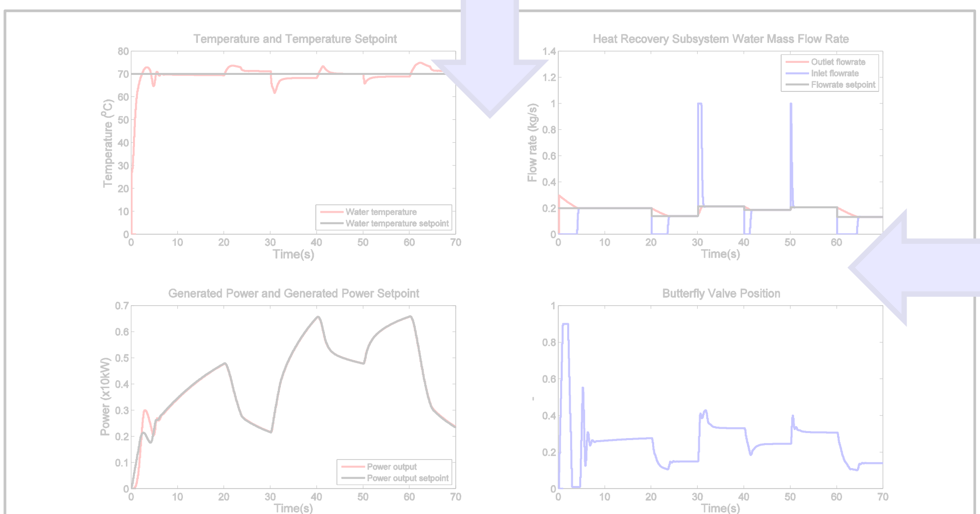


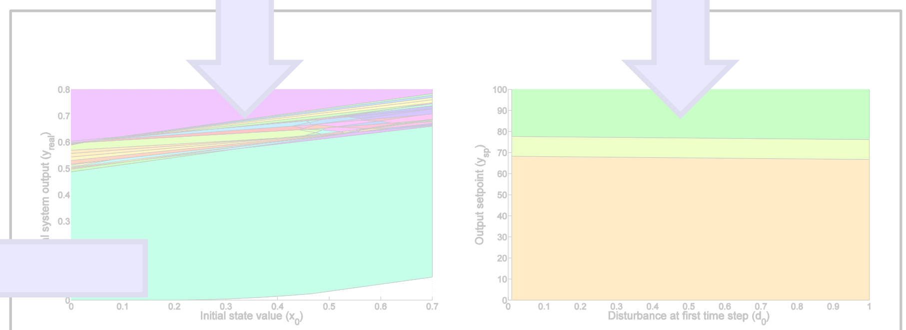
mp-P receding horizon policies with optimal design and scheduling dependence

PAROC Framework



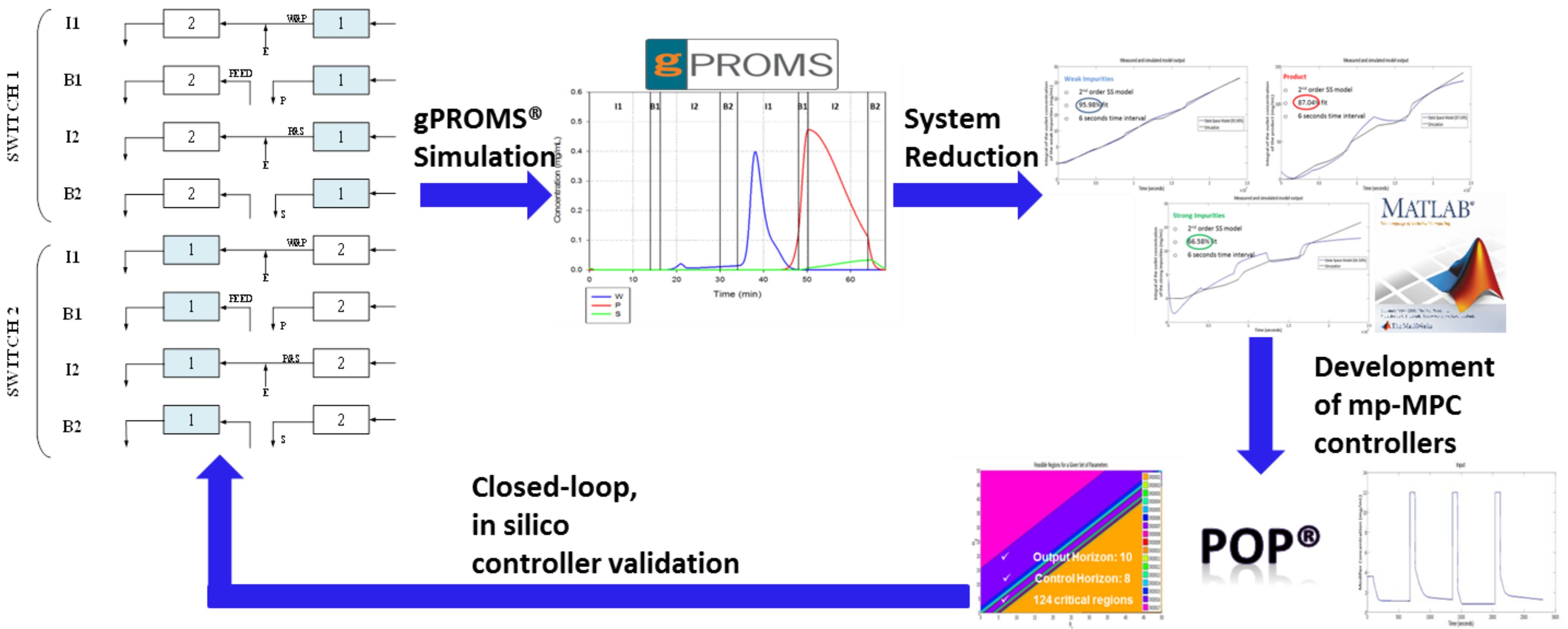


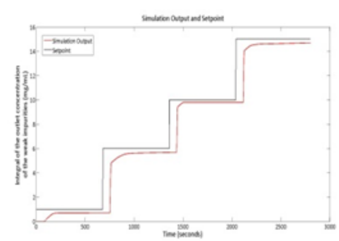




mp-P receding horizon policies with optimal design and scheduling dependence

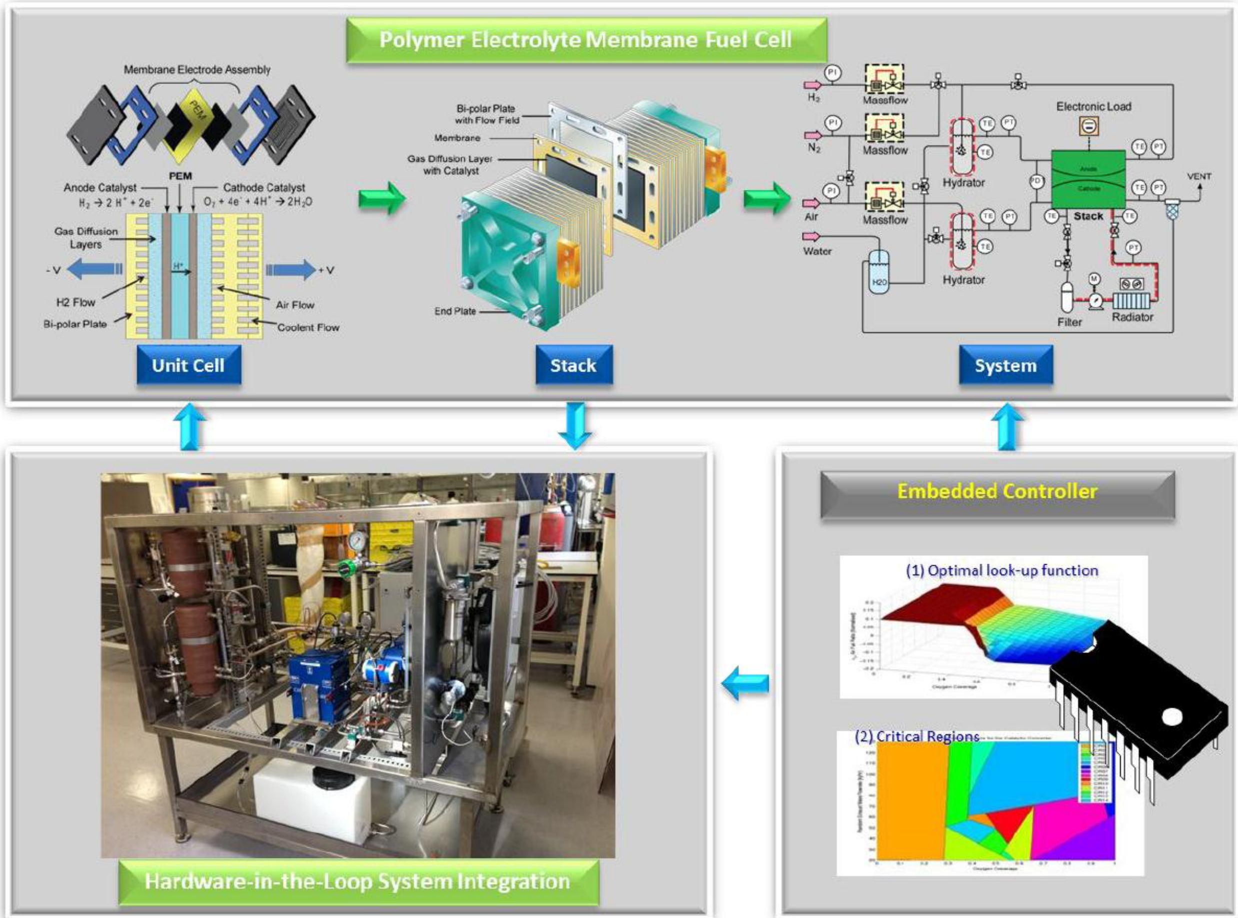
PAROC Framework – *MCSGP Process*





- **Semi-continuous** chromatographic separation designed for **biopharmaceutical** applications.
- **Periodic**, **nonlinear** system.
- Use of **software interoperability** for the controller development.
- Obtain high purity and yield under **continuous** operation.

PAROC Framework – *PEMFC Process*

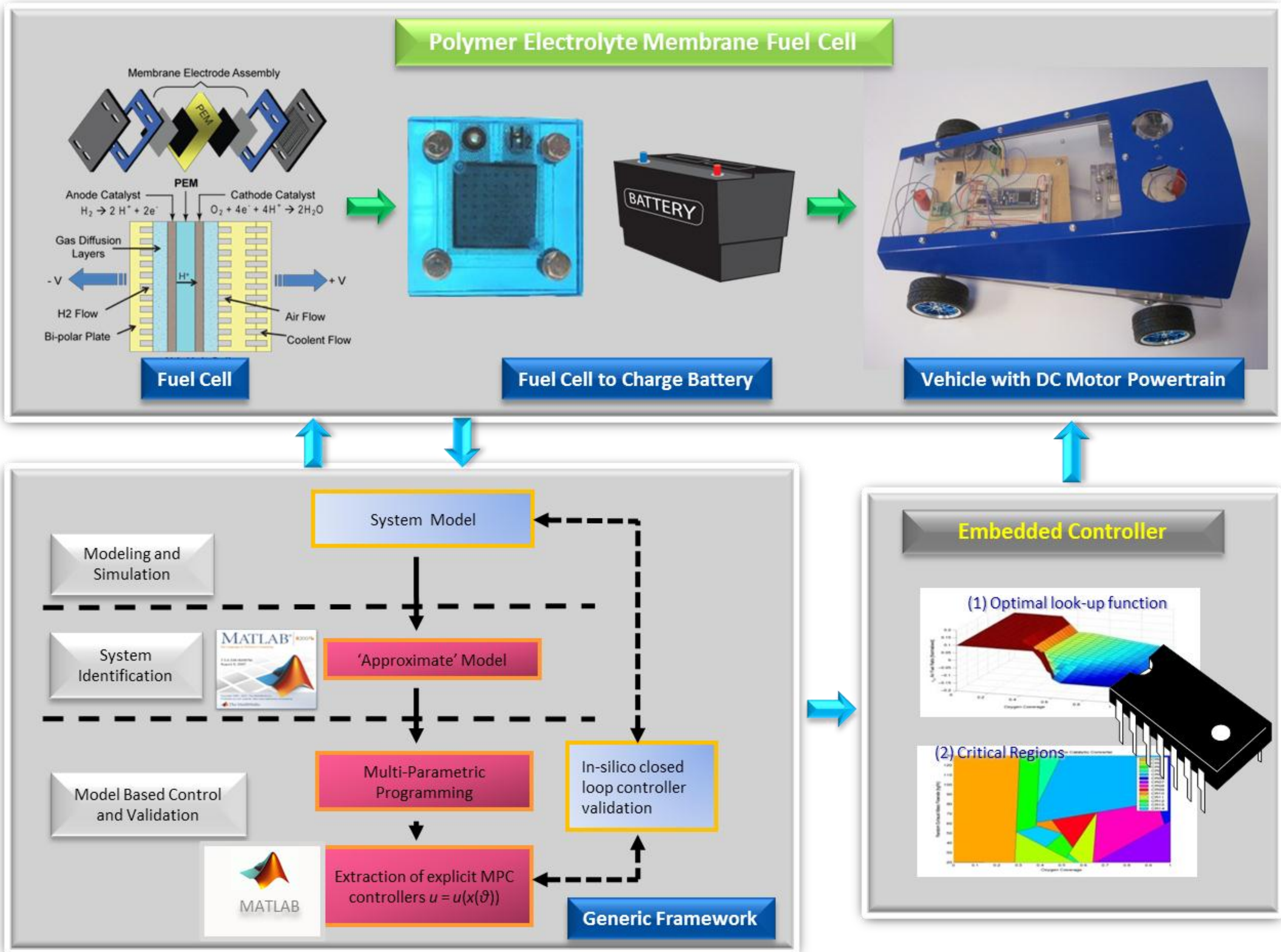


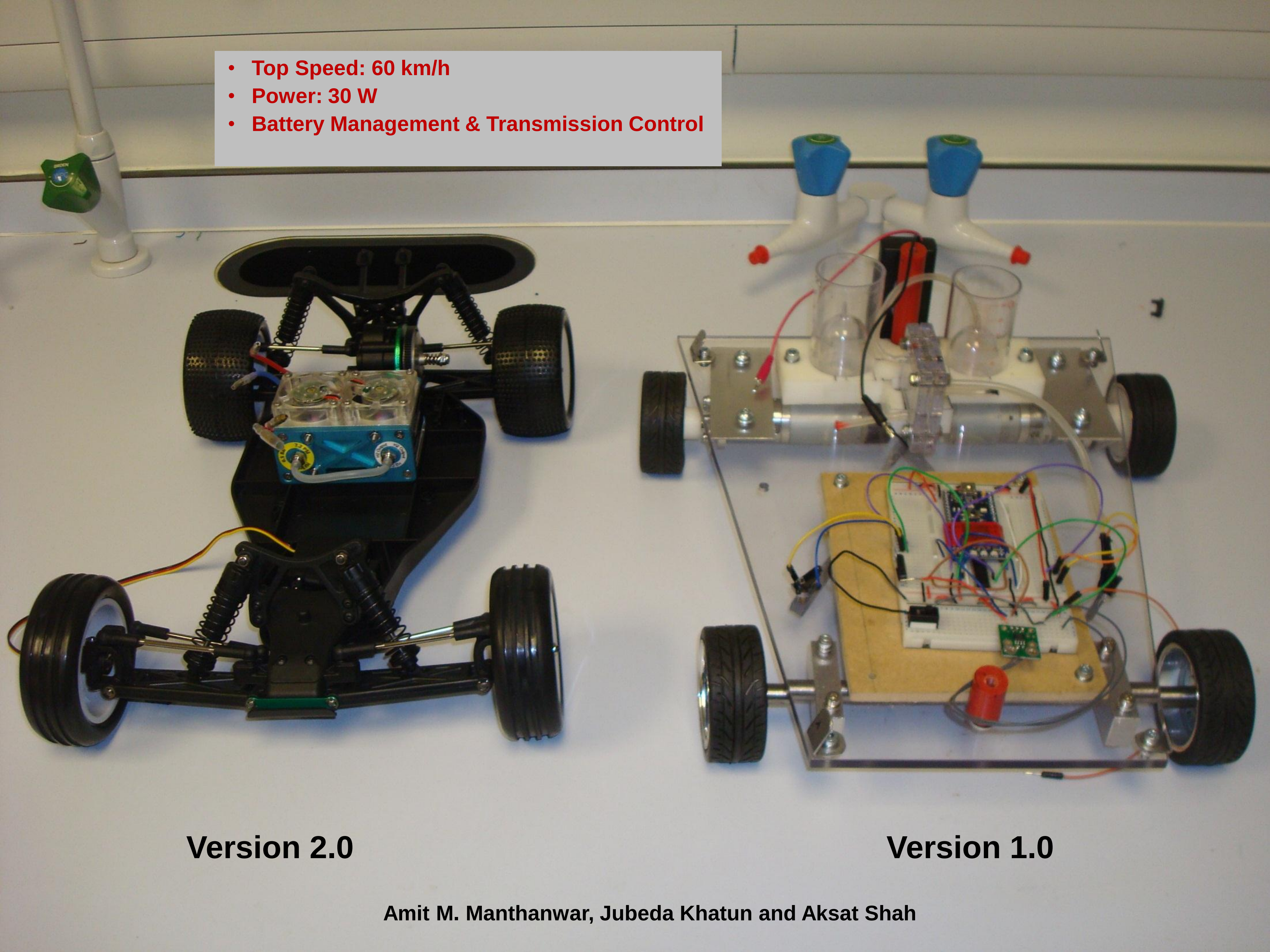
Control critical for the robust operation of the fuel cell system— essential for:

- uninterruptable power demand under load variations
- high efficiency and performance under uncertainty
- reliability and longevity

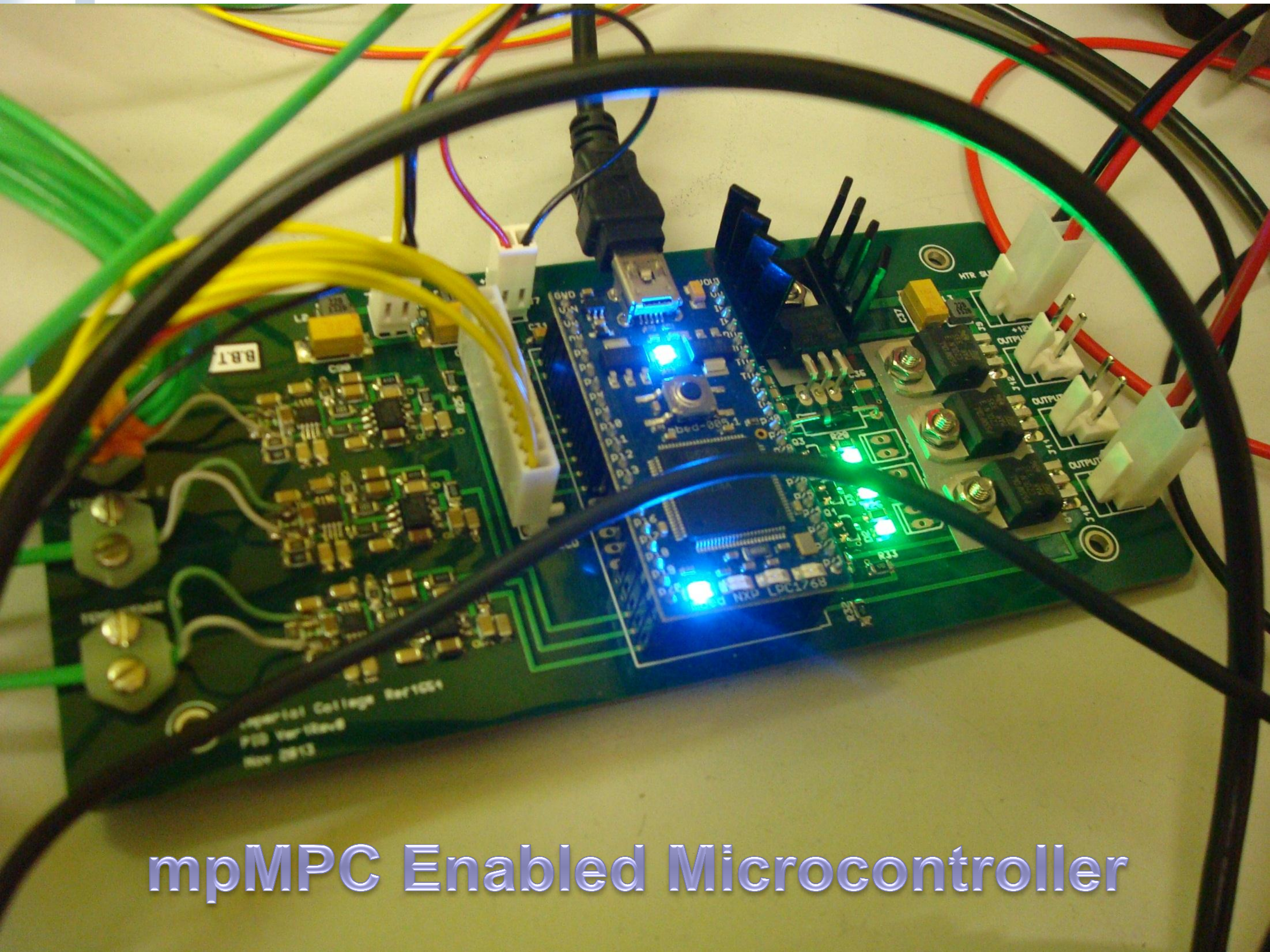

Fully Automated Pilot Plant

PAROC Framework - *Mini Vehicle*



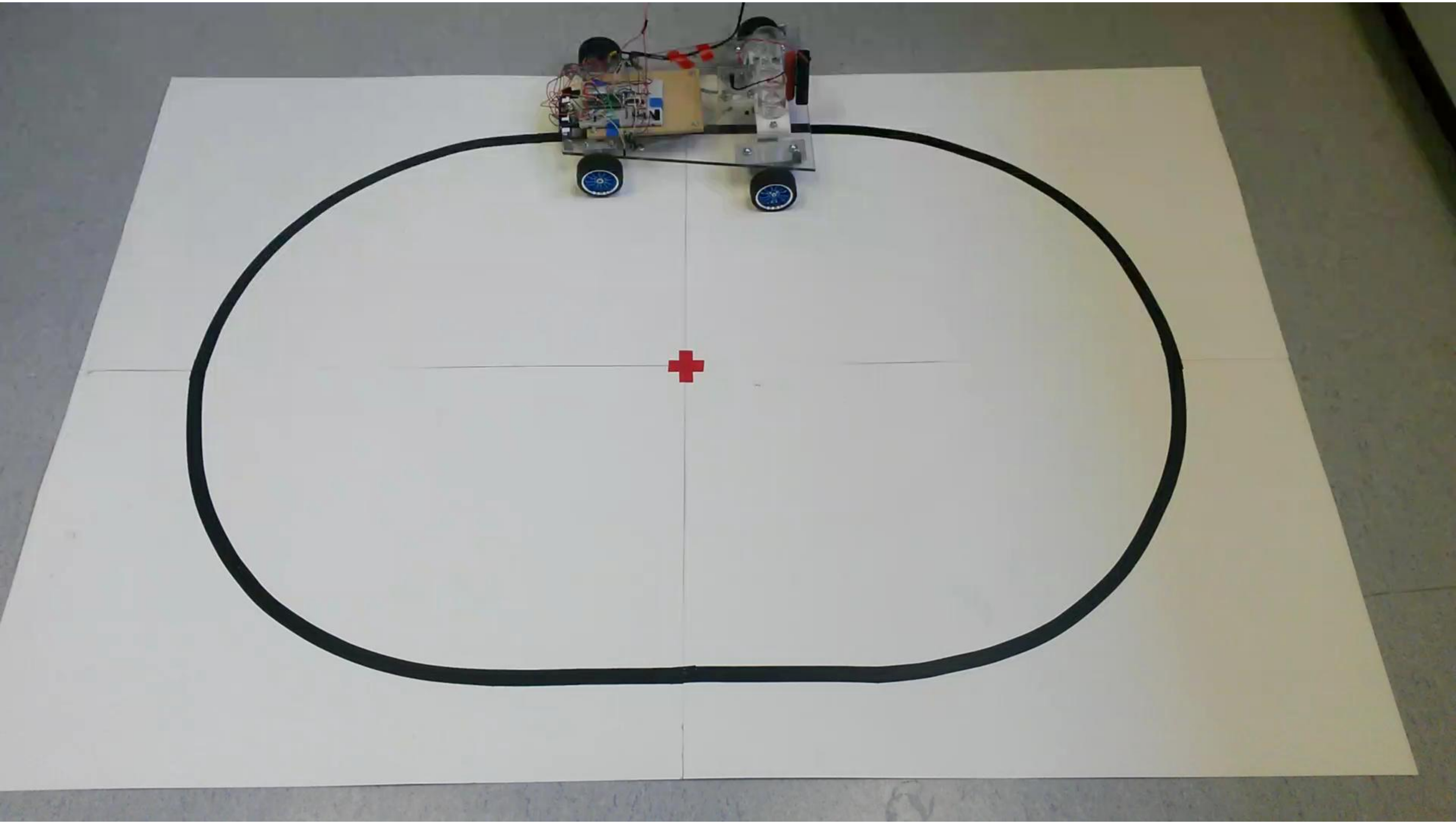


Amit M. Manthanwar, Jubeda Khatun and Aksat Shah



mpMPC Enabled Microcontroller

Demonstration of mp-MPC



To our parents we owe our being

To our teachers we owe our well being

(Alexander the Great)

THANK YOU IGNACIO!

… for your inspirational Academic Leadership, Pioneering Vision & continuous Quest for Excellence

To our parents we owe our being

To our teachers we owe our well being

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& HAPPY 65th Birthday!

Centre for Process Systems Engineering

Imperial College London

Advances in multi-parametric programming & explicit Model Predictive Control

Stratos Pistikopoulos

Session in honor of Professor Ignacio E. Grossmann